On $\tau^{*}m_{wg}$ -Continuous Multifunctions in Topological spaces

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ABSTRACT:

In this paper, we introduced and studied basic properties of weaker form of multifunctions such as upper and lower $\tau^{*}m_{wg}$-continuous multifunctions. We obtain some of its characterizations with totally $\tau^{*}m_{wg}$-closed graph and strongly $\tau^{*}m_{wg}$-closed graph in Minimal Structures.

KEYWORDS: $\tau^{*}m_{wg}$ -continuous, $\tau^{*}m_{wg}$-connected, $\tau^{*}m_{wg}$-compact, Totally m_{wg}-closed graph, and Strongly m_{wg}-closed graph.

INTRODUCTION

The concept of minimal structure (briefly m-structure) was introduced by V. Popa and T. Noiri [10] in 2000. Also they introduced the notions of m_X-open sets and m_X-closed sets and characterize those sets using m_X-closure and m_X-operators, respectively. T. Noiri and V. Popa [7] obtained the definition and characterizations of separation axioms by using the concept of minimal structure.

Csa´sza´r [3] introduced the concept of generalized neighborhood systems and generalized topological spaces. He also introduced the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In particular, he investigated characterizations for the generalized continuous function by using a closure operator defined on generalized neighborhood systems. Moreover he studied the simplest separation axioms for generalized topologies in [2]. Nagaveni *et al.,* defined weakly generalized closed sets [8] and mg-continuous functions [9] in Minimal structures, and D. Sheeba and N. Nagaveni [11] defined Multifunction with topological closed Graphs.

A multifunction [1] $F: X \rightarrow Y$ is a point to set correspondence and we always assume that $F(x) \neq \emptyset$ for every point $x \in X$. For a multifunction F, the upper and lower set V of Y will be denoted by $F^+(V)$ and $F^-(V)$ respectively, that is, $F^+(V) = \{x \in X: F(x) \subset V\}$ and $F^-(V) = \{x \in X: F(x) \cap V \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X: y \in F(x)\}$ for each point $y \in Y$. For each $A \subset X, F(A) = \cup_{x \in A} F(x)$. Then F is said to be a surjection if $F(X) = Y$ or equivalently, if for each $y \in Y$ there exists a $x \in X$ such that $y \in F(x)$. The graph multifunction $G_F: (X, \tau) \rightarrow (X \times Y, \tau \times \sigma)$ of F is defined by $G_F(x) = \{\{x\} \times F(x)\}$ for each $x \in X$. Graph of F (ie.) $G(F) = \{(x, y) / x \in X, y \in F(x)\}$. We say that F has a closed graph if $G(F)$ is closed in $(X \times Y, \tau \times \sigma)$. Throughout this paper $(X, \tau^* m_X)$ and (Y, m_Y) are denoted by minimal structure (briefly m-space) and τ^* is defined by τ^*={G:cl*(G^c)=G^c}

PRELIMINARIES

Definition: 2.1

Let X be a non empty set and $P(X)$ the power set of X. A subfamily m_X of P(X) is called a minimal structure (briefly m-structure) on X if $\emptyset \in m_X$ and X $\in m_X$. By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and call it an m-space. Each member of m_X is said to be m_X-open and the complement of an m_X-open set is said to be m_X-closed. **[6]**

Definition: 2.2
Let X be a nonempty set and m_X an m-structure on X. For subset A of X, the m_X-closure of A and the m_X -interior of A are defined as follows:
(i) m_X -Cl(A) = ∩{F : A⊂ F, X - F ∈m_X},
(ii) m_X -Int(A) = ∪{U : U ⊂ A, U ∈m_X}.[4]

Lemma: 2.3
Let (X, m_X) be a space with minimal structure, let A be a subset of X and $x \in X$. Then $x \in m_X\text{-}Cl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing the point x. [6]

Remark: 2.4

Let (X, τ) be a topological space. Then the families $\tau, SO(X), PO(X), \alpha(X), \beta(X), SR(X))$ are all m-structures on X.[6]

Remark: 2.5

Let (X, τ)be a topological space and A *be a* subset of X. If m_X $=\tau(resp. SO(X), PO(X), \alpha(X), \beta(X), SR(X))$, then we have

(i) $m_X - Cl(A) = Cl(A)$ $(resp. sCl(A), pCl(A), \alpha Cl(A), \beta Cl(A), s_\theta Cl(A))$,
(ii) $m_X - Int(A) = Int(A)$ $(resp. sInt(A), pInt(A), \alpha Int(A), \beta Int(A), s_\theta Int(A))$.[6]

Lemma: 2.6

Let X be a nonempty set and m_X a minimal structure on X. For subsets A and B of X, the following hold:
(i) m_X -Cl(X - A) = X - (m_X -Int(A)) and m_X -Int(X - A) = X - (m_X -Cl(A)),

(ii) If (X - A) ∈ m_X, then m_X -Cl(A) = A and if A ∈ m_X, then m_X -Int(A) = A,
(iii) m_X -Cl(∅) =∅, m_X-Cl(X) = X, m_X -Int(∅) = ∅ and m_X -Int(X) = X,
(iv) If A ⊂ B, then m_X-Cl(A) ⊂ m_X -Cl(B) and m_X - Int(A) ⊂ m_X - Int(B),
(v) A ⊂ m_X -Cl(A) and m_X -Int(A) ⊂ A,
(vi) m_X -Cl(m_X -Cl(A)) = m_X -Cl(A) and m_XNX -Int(m_X -Int(A)) = m_X -Int(A). [4]

Definition: 2.7

A subset A of a m-space (X, m_X) is said to be minimal weakly generalized closed (briefly m_{wg} -closed) sets if m_X - Cl (m_X - Int(A)) ⊂ U whenever A ⊂ U and U is open in m_X. The complement of m_{wg} -closed set is said to be m_{wg} -open set. The family of all m_{wg} -open (resp. m_{wg} -closed) sets is denoted by m_X -WGO(X) (resp. m_X -WGC(X)). We define, m_X - $WGO(X, x) = \{V \in m_X\text{-}WGO(X) \,/ x \in V\}$ for $x \in m_X$. [8]

Lemma: 2.8

For a multifunction F: (X, m_X) → (Y, m_Y) following hold:

(i) G_F^+(A × B) = A ∩ F^+(B),
(ii) G_F^-(A× B) = A ∩ F^-(B), for any subsets $A \subseteq X$ *and* $B \subseteq Y$. **[6]**

Definition: 2.9

A m-space(X,m_X) is said to be

(i) m- Haussdroff if for any distinct points x, y there exists U, V ∈m_X such that $x \in U$, $y \in V$ *and* $U \cap V = \emptyset$.
(ii) m- Urysohn if for any distinct points x, y there exists U, V ∈ m_X such that $x \in U, y \in V$ and m_X -Cl(U) ∩ m_X -Cl(V) = ∅.
(iii) m- compact if every cover of X by m_{wg} -open sets has a finite sub cover. **[5]**

Definition: 2.10

A m-space (X, m_X) is called

(i)m_{wg}- Haussdroff space (i.e. m_{wg}-T2 space) if for every pair of distinct points x, y in X there exists disjoint m_{wg} -open sets $U \in X$ and $V \in X$ containing x and y respectively.

(ii) m_{wg}--normal if for each pair of non empty disjoint m-closed sets can be separated by disjoint m_{wg} -open sets.
(iii)m_{wg}-regular if for each m_{wg}-closed set F of X and each $x \notin F$, there exist disjoint m_{wg} -open sets U and V such that $F \subset U$ and $x \in V$. **[4]**

Definition:2.11

A graph of a multifunction F: (X, m_X) → (Y, m_Y) is said to be totally m_{wg}-closed if for each (x, y) ∈ (X× Y)-G(F), there exists U∈ m_X-WGCO(X,x)and V∈ m_Y-O(Y,y)such that(U× V)- ∩G(F)=∅.[11]

Definition:2.12

For a multifunction F: (X, m_X) → (Y, m_Y),the graph $G(F) = \{(x, F(x)): x \in X\}$ is said to be strongly m_{wg}-closed if for each $(x, y) \in (X\times Y)$-G(F),there exists U∈ m_X-WGO(X,x)and V∈ m_Y-WGO(Y,y)such that (U × m_Y-Cl(V)∩ G(F)=∅.[11]

3. UPPER AND LOWER $\tau^* m_{wg}$ -CONTINUOUS MULTIFUNCTION

In this section, we defined and investigated a new weaker form of multifunction such as upper and lower $\tau^* m_{wg}$ -continuous multifunction in Minimal Structures.

Definition: 3.1

A multifunction F: (X, $\tau^* m_X$) → (Y, m_Y) is called

(i) upper $\tau^* m_{wg}$ -continuous (briefly, u. $\tau^* m_{wg}$ -c.) at a point $x \in X$ if for each m-open subset V of Y with $F(x) \subseteq V$, there exists an $\tau^* m_{wg}$ -open set U containing x such that $F(U) \subseteq V$.

(ii) lower $\tau^* m_{wg}$ -continuous (briefly, l. $\tau^* m_{wg}$ -c) at a point $x \in X$ if for each m-open subset V of Y with $F(x) \cap V \neq \emptyset$, there exists an $\tau^* m_{wg}$ -open set U containing x such that $F(y) \cap V \neq \emptyset$, for every point $y \in U$.

Remark: 3.2

From the following examples, it is clear that upper $\tau^* m_{wg}$ -continuous and lower $\tau^* m_{wg}$ -continuous are independent of each other.

Example:3.3

Let X={a,b,c} and Y= {1,2,3} be a topology with the minimal structures $\tau^* m_X$={X,∅,$\{a,b\},\{a\},\{b\}\}$ *and* $m_Y = \{Y, \emptyset, \{3\}, \{1,3\}\}$. Let F: (X, $\tau^* m_X$) → (Y, m_Y) be a multifunction defined by $F(a) = \{3\}, F(b) = \{1,3\}, F(c) = \{2\}$. Then F is upper $\tau^* m_{wg}$ -continuous.

Example:3.4

Let X={a,b,c} and Y= {1,2,3} be a topology with the minimal structures $\tau^* m_X$={X,∅} *and* m_Y={Y, ∅,{2,3}}. Let F: (X, $\tau^* m_X$) → (Y, m_Y) be a multifunction defined by F(a)={1},F(b)={3},F(c)={1,2}. Then F is lower $\tau^* m_{wg}$ -continuous, but it is not upper $\tau^* m_{wg}$ -continuous.

Theorem: 3.5

Let F: (X, $\tau^* m_X$)) → (Y, m_Y) be a multifunction. Then the following statements are equivalent.

(i) F: (X, $\tau^* m_X$) → (Y, m_Y) is an upper $\tau^* m_{wg}$ -continuous.
(ii) F^+(V) ∈ $\tau^* m_X$- WGO(X) for each V ∈ $\tau^* m_X$- O(Y).
(iii) F^- (V) ∈ $\tau^* m_X$- WGC(X) for each V ∈ $\tau^* m_X$- O(Y).

Proof:

(i) ⇔ (ii) Let V be a m-open subset set of m_Y and $x \in F^+$(V). Since F: (X, $\tau^* m_X$) → (Y,m_Y) is an upper $\tau^* m_{wg}$ -continuous, there exists U ∈ $\tau^* m_X$- $WGO(X, x)$ such that $F(U) \subseteq V$. Hence, F^+(V) is $\tau^* m_{wg}$ -open in X.

(ii) ⇔ (iii) It follows that $F^+(Y \setminus V) = X \setminus F^-(V)$ for every subset V of Y.

Theorem: 3.6

Let F: (X, $\tau^* m_X$) → (Y, m_Y) be a multifunction. Then the following statements are equivalent.

(i) F: (X, $\tau^* m_X$) → (Y, m_Y) is a lower $\tau^* m_{wg}$ -continuous.
(ii) F^- (V) ∈ $\tau^* m_X$- WGO(X) for each V ∈ $\tau^* m_X$- O(Y).
(iii) F^+ (V) ∈ $\tau^* m_X$- WGC(X) for each V ∈ $\tau^* m_X$- O(Y).

The proof follows from the definitions and properties.

Theorem 3.7

For a multifunction F : (X, $\tau^* m_X$) →(Y, m_Y)),following properties are equivalent:

(i) F is upper $\tau^* m_{wg}$ -continuous;
(ii) F^+ (V) = $\tau^* m_X$-Int(F^+ (V)) for every V ∈ m_Y ;
(iii) F^-(K) = $\tau^* m_X$-Cl(F^- (K)) for every m_Y -closed set K;
(iv) $\tau^* m_X$ -Cl(F^-((B)) ⊂ F^- (m_Y -Cl(B)) for every subset B of Y;
(v) F^+(m_Y -Int(B)) ⊂ $\tau^* m_X$-Int(F^+ (B)) for every subset B of Y.

Proof.

(i) ⇒ (ii): Let V ∈ m_Y and $x \in F^+$(V). Then F(x) ⊂ V . There exists $\tau^* m_X$ containing x such that $F(U) \subset V$. Then $x \in U \subset F^+$(V). So that $x \in \tau^* m_X$-Int(F^+(V)). This shows that F^+(V)⊂ $\tau^* m_X$- Int(F^+(V)). By Lemma 2.6(v), we have $\tau^* m_X$-Int(F^+(V)) ⊂ F^+(V). Hence, F^+(V) = $\tau^* m_X$-Int(F^+(V)).

(ii) ⇒ (iii): Let K be any m_Y-closed set. Since Y-K ∈ $\tau^* m_X$, by Lemma 2.6(ii) we have X-F^-(K) = F^+(Y-K) = $\tau^* m_X$-Int(F^+(Y-K)) = $\tau^* m_X$-Int(X-F^-(K))= X-$\tau^* m_X$-Cl(F^-(K)). Then, we obtain $\tau^* m_X$- Cl(F^-(K)) = F^-(K).

(iii) ⇒ (iv): Let B be any subset of Y. By Lemma 2.6(iv), m_Y -Cl(B) is m_Y -closed. By Lemma 2.6(iv), we have F^-(B) ⊂ F^-(m_Y-Cl(B)) = $\tau^* m_X$-Cl(F^-(m_Y -Cl(B)) and $\tau^* m_X$-Cl(F^-(B)) ⊂ F^-(m_Y -Cl(B)).

(iv)⇒ (v): Let B be any subset of Y. Then by Lemma 2.6(i) we have X − $\tau^* m_X$-Int(F^+(B))) = $\tau^* m_X$-Cl(X − F^+(B)) = $\tau^* m_X$-Cl(F^-(Y−B)) ⊂ F^-(m_Y -Cl(Y−B)= F^-(Y− m_Y -Int(B)) = X − F^+(m_Y -Int(B)). We obtain F^+(m_Y -Int(B)) ⊂ $\tau^* m_X$-Int(F^+(B)).

(v)⇒ (i): Let $x \in X$ and $V \in m_Y$ containing $F(x)$. Then $x \in F^+$(V) = F^+(m_Y-Int(V)) ⊂ $\tau^* m_X$-Int(F^+(V)). There exists U ∈ $\tau^* m_X$ containing x such that U ⊂ F^+(V); hence $F(U) \subset V$. This shows that F is upper $\tau^* m_{wg}$ -continuous.

Theorem 3.8

For a multifunction F: (X, $\tau^* m_X$)→(Y, m_Y) following properties are equivalent:

(i) F is lower $\tau^* m_{wg}$ -continuous;

(ii) F^-(V) = $\tau^* m_X$-Int(F^-(V)) for every V ∈ m_Y ;

(iii) F^+ (K) = $\tau^* m_X$-Cl(F^+(K)) for every m_Y -closed set K;

(iv) $\tau^* m_X$-Cl(F^+(B)) ⊂ F^+(m_Y -Cl(B)) for every subset B of Y;

(v) F ($\tau^* m_X$-Cl(A)) ⊂ m_Y -Cl(F(A)) for every subset A of X;

(vi) F^-(m_Y -Int(B)) ⊂ $\tau^* m_X$-Int(F^-(B)) for every subset B of Y.

Proof.

The proof (i)⇒(ii) ,(ii)⇒(iii) , (iii)⇒(iv) are similar to the above theorem.

(iv)⇒(v): Let A be any subset of X. By (iv), we have $\tau^* m_X$-Cl(A) ⊂ $\tau^* m_X$- Cl(F^+(F(A)))⊂ F^+(m_Y -Cl(F(A))) and F ($\tau^* m_X$-Cl(A)) ⊂ m_Y-Cl(F (A)).

(v)⇒(vi): Let B be any subset of Y. By(v), we have F ($\tau^* m_X$-Cl(F^+(Y−B)))⊂ m_Y -Cl(F (F^+(Y− B))) ⊂ m_Y -Cl(Y− B)) =Y−m_Y -Int(B) and F ($\tau^* m_X$-Cl(F^+ (Y− B))) = F ($\tau^* m_X$-Cl(X− F^-(B))) = F (X− $\tau^* m_X$-Int(F^-(B))). This shows that F^-(m_Y-Int(B)) ⊂ $\tau^* m_X$-Int(F^-(B)).

4. CHARACTERIZATION OF UPPER AND LOWER $\tau^* m_{wg}$ -CONTINUOUS MULTIFUNCTION WITH GRAPH OF MULTIFUNCTION

We obtain some of upper and lower $\tau^* m_{wg}$ -continuous characterizations with graph of multifunction, totally $\tau^* m_{wg}$-closed graph and strongly $\tau^* m_{wg}$-closed graph in Minimal Structures. Recall that, if $F: X \rightarrow Y$ is a Multifunction, then the graph of F is the subset $\cup \{\{x\} \times f(x): x \in X\}\ of\ X \times Y$. Graph of F is denoted by $G(F)$.

Theorem: 4.1

Let $\tau^* m_X$ and m_Y be m-spaces and let F: (X, $\tau^* m_X$) → (Y, m_Y) be multifunction. If the graph function $G_F: X \rightarrow X \times Y$ is upper $\tau^* m_{wg}$-continuous multifunction, then F is upper $\tau^* m_{wg}$-continuous multifunction.

Proof:

Suppose that G_F is upper $\tau^* m_{wg}$-continuous. Let $x \in X$ and W be any m-open set of m_Y such that $F(x) \subset V$. Then $G_F(x) \subset (X \times V)$ and $X \times V$ is m-open set in $X \times Y$. Since G_F is upper $\tau^* m_{wg}$ -continuous, there is an $\tau^* m_{wg}$-open set U with $x \in U$ such that $G_F(U) \subset X \times V$. By Lemma 2.8(i), U$\subset$ G^+(X × V) = X∩ F^+(V) = F^+(V) and $F(x) \subset V$. So F is upper $\tau^* m_{wg}$ - continuous at $x \in X$.

Theorem: 4.2

A multifunction F: (X, $\tau^* m_X$) → (Y, m_Y) is lower $\tau^* m_{wg}$-continuous multifunction if and only if the graph multifunction G_F is lower $\tau^* m_{wg}$-continuous.

Proof:

Suppose that F is lower $\tau^* m_{wg}$-continuous multifunction. Let $x \in X$ and W be any $\tau^* m_{wg}$-open set of $X \times Y$ such that $x \in G^-$(W). Since $W \cap \{\{x\} \times F(x)\} \neq \emptyset$, there exists $y \in F(x)$ such that $(x, y) \in W$ and hence (x, y) ∈ U × V ⊆ W for some $\tau^* m_{wg}$-open sets of U and V of X and Y, respectively. Since $F(x) \cap V \neq \emptyset$, there exists G ∈ $\tau^* m_X$-WGO(X, x) such that G ⊆ F(V). By Lemma 2.8(ii), $U \cap G \subseteq U \cap F^-$(V) = G^-(U×V) ⊆ G^-(W). So, we obtain $x \in U \cap G \in \tau^* m_X - WGO(X, x)$ and hence G_F is lower $\tau^* m_{wg}$-continuous. Let us assume that G_F is lower $\tau^* m_{wg}$-continuous. Let $x \in X$ and W be any m-open set of m_Y such that $x \in F^-$(V). Then X×V is $\tau^* m_{wg}$-open in X × Y and $G_F(x) \cap (X \times V) = (\{x\} \times F(x)) \cap (X \times V) = \{x\} \times (F(x) \cap V) \neq \emptyset$. Since G_F is lower $\tau^* m_{wg}$ -continuous, there exists an $\tau^* m_{wg}$ -open set U containing x such that U ⊆ G^-(X × V). By Lemma 2.8(ii) we have U ⊆ F^-(V). This shows that F is lower $\tau^* m_{wg}$ -continuous multifunction.

Definition: 4.3

A nonempty set X with minimal structures $\tau^* m_X$ is said to be $\tau^* m_{wg}$-compact if every cover of X by $\tau^* m_{wg}$ -open sets has a finite subcover. A subset K of a nonempty set X with a minimal structure $\tau^* m_X$ is said to be $\tau^* m_{wg}$ -compact if every cover of K by $\tau^* m_{wg}$ -open sets has a finite subcover.

Theorem: 4.4

If F: (X, $\tau^* m_X$) → (Y,m_Y) is an upper $\tau^* m_{wg}$ -continuous multifunction and $F(x)$ is $\tau^* m_{wg}$ -compact, then the graph multifunction G_F is upper $\tau^* m_{wg}$ -continuous.

Proof:

Let $x \in X$ and W be any $\tau^* m_{wg}$ -open sets of $X \times Y$ containing $G_F(x)$. For each $y \in F(x)$, there exist $\tau^* m_{wg}$ -open sets $U(y) \subseteq X$ and $V(y) \subseteq Y$ such that $(x, y) \in U(y) \times V(y) \subseteq W$. The family of $\{V(y) : y \in F(x)\}$ is an $\tau^* m_{wg}$ -open cover of F(x). Since $F(x)$ is $\tau^* m_{wg}$ -compact, it follows that there exists a finite number of points, says $y_1, y_2, y_3, \dots y_n$ in F(x) such that $F(x) \subseteq \{V(y_i): i = 1, 2, \dots, n\}$. $Take\ U = \cap \{U(yi) : i = 1, 2, \dots, n\}$ and $V = \cap \{V(yi) : i = 1, 2, \dots, n\}$. Then U and V are $\tau^* m_{wg}$-open sets in X and Y, respectively, and $\{x\} \times F(x) \subseteq U \times V \subseteq W$.since F is an upper $\tau^* m_{wg}$-continuous, there exist $U_0 \in \tau^* m_X$ -WGO(X, x) such that $F(U_0) \subseteq V$. By Lemma 2.8(i), we have $U \cap U_0 \subseteq U \cap F^+$(V) = G^+(U × V)∈ G^+(W). Then, we obtain U∩$U_0 \in \tau^* m_X$-WGO(X, x) and $G_F(U \cap U_0) \subseteq W$. Hence G_F is upper $\tau^* m_{wg}$ -continuous.

Definition: 4.5

A space (X, $\tau^* m_X$) is said to be $\tau^* m_{wg}$ -connected if it cannot be written as the union of two nonempty disjoint $\tau^* m_{wg}$ -open sets.

Theorem: 4.6

Let F: (X, $\tau^* m_X$) → (Y, m_Y) be a multifunction and X be a $\tau^* m_{wg}$ -connected space. If the graph multifunction G_F is upper $\tau^* m_{wg}$ -continuous, then F is upper $\tau^* m_{wg}$ -continuous.

Proof:

Let $x \in X$ and V be any open subset of Y containing $F(x)$. Since $X \times V$ is a $\tau^* m_{wg}$ -open set of X × Y and $G_F(U) \subset X \times V$, there exist a $\tau^* m_{wg}$ -open set U containing x such that $G_F(U) \subset X \times V$. By the Lemma 2.8(i) , we have U⊂ G^+(X × V) = F^+(V) and $F(U) \subseteq V$. Thus, F is upper $\tau^* m_{wg}$ -continuous.

Theorem: 4.7

Let F: (X, $\tau^* m_X$) → (Y, m_Y) be a multifunction and X be a $\tau^* m_{wg}$ -connected space. If the graph multifunction G_F is lower $\tau^* m_{wg}$ -continuous, then F is lower $\tau^* m_{wg}$ -continuous.

Proof:

Let $x \in X$ and V be any open subset of Y containing $F(x)$. Since $X \times V$ is a $\tau^* m_{wg}$ -open set of X × Y and $G_F(U) \cap X \times V \neq \emptyset$, there exist a $\tau^* m_{wg}$ -open set U containing x such that

$G_F(U) \cap X \times V \neq \emptyset$. By the Lemma 2.8(ii), we have U ⊂ G^+(X × V) = F^-(V) and $F(U) \cap V \neq \emptyset$. Thus, F is lower $\tau^* m_{wg}$-continuous.

Definition: 4.8

A graph of a multifunction F: (X, $\tau^* m_X$) → (Y, m_Y) is said to be $\tau^* m_{wg}$-closed if for each $(x, y) \in$ (X × Y) - G(F), there exists U ∈ $\tau^* m_X$-WGO(X, x) and V ∈ m_Y-WGO(Y, y) such $t\ \ at\ (U \times V) \cap G(F) = \emptyset$.

Lemma: 4.9

A multifunction F: (X, $\tau^* m_X$)→(Y, m_Y) has a $\tau^* m_{wg}$-closed if and only if for each $(x, y) \in$ (X × Y) - G(F), there exists U ∈ $\tau^* m_X$ -WGO(X, x) and V ∈ $\tau^* m_X$ -WGO(Y, y) such that $F(U) \cap V \neq \emptyset$.

The proof is obvious.

Theorem: 4.10

If F: (X, $\tau^* m_X$)) → (Y, m_Y) is a point closed upper $\tau^* m_{wg}$-continuous multifunction into a $\tau^* m_{wg}$ -regular space, then F has a $\tau^* m_{wg}$-closed graph.

Proof:

Suppose $(x, y) \in (X \times Y) - G(F). T\ \ en\ y \notin F(x)$. Thus there are disjoint m -open sets U,V ⊂ Y such that $F(x) \subset U$ and $y \in V$. Since F is upper $\tau^* m_{wg}$-continuous , there is an $\tau^* m_{wg}$- open set W ⊂X containing x, such that $F(W) \subset U$. Thus $(x, y) \in W \times V\ and\ (W \times V) \cap G(F) = \emptyset$. Hence, $G(F)$ is $\tau^* m_{wg}$-closed graph.

Theorem: 4.11

Let F: (X, $\tau^* m_X$) → (Y, m_Y) be a multifunction from a space X into a $\tau^* m_{wg}$- compact space Y. If G(F) is $\tau^* m_{wg}$-closed, then F is upper $\tau^* m_{wg}$-continuous.

Proof:

Suppose that F is not upper $\tau^* m_{wg}$-continuous. Then there exists a nonempty m-open subset C of Y such that F^-(C) is not $\tau^* m_{wg}$ -open in X. We may assume $F^-(C) \neq \emptyset$. Then there exists a point $x_0 \notin F^-$(C). Hence for each point y ∈ C, we have $(x_0, y) \notin G(F)$. Since F has a $\tau^* m_{wg}$-closed, there are m_{wg}-open subsets U(y) and V(y) containing x_0 and y, respectively such that $(U(y) \times V(y)) \cap G(F) = \emptyset$ Then $\{Y \backslash C\} \cup \{V(y) : y \in C\}$ is a $\tau^* m_{wg}$-open cover of Y, and it has a sub cover {Y\C} ∪ {V(yi) : y_i ∈ C, 1 ≤ i ≤ n}. Let U = $\bigcap_{i=1}^{n}$ U(y_i) and $V = \bigcup_{i=1}^{n}$ V(y_i). It is easy to verify that C ⊂ V and (U × V) ∩ G(F) = ∅. Since U is $\tau^* m_{wg}$-neighbourhood of x_0, U ∩ F^-(C) ≠ ∅. It follows that ∅ ≠ (U × C) ∩ G(F) ⊂ (U × V) ∩ G(F). Which is a contradiction. Then, G(F) is $\tau^* m_{wg}$-closed graph.

Lemma:4.12

A multifunction F: (X,$\tau^* m_X$)→(Y,m_Y) has a totally m_{wg}-closed if and only if for each $(x,y) \in (X \times Y) - G(F)$, there exists U∈ m_X-WGCO(X,x)and V∈ m_Y-O(Y,y)such that F(U)∩ V=∅.

The proof is obvious.

Theorem: 4.13

Let F: (X, $\tau^* m_X$) → (Y, m_Y) be a multifunction from a space X into a m-compact space Y. If G(F) is totally m_{wg}-closed , then F is upper $\tau^* m_{wg}$-continuous.

Proof:

Suppose that F is not upper $\tau^* m_{wg}$-continuous. Then there exists a nonempty m-open subset C of Y such that F^-(C) is not $\tau^* m_{wg}$-open in X. We may assume $F^-(C) \neq \emptyset$. Then there exists a point x_0 ∉ F^-(C). Hence for each point y ∈ C, we have $(x_0, y) \notin G(F)$. Since F has a totally m_{wg}-closed, there are m_{wg}-open subsets U(y) and m-open subsets V(y) containing x_0 and y, respectively such that $(U(y) \times V(y)) \cap G(F) = \emptyset$. Then $\{Y \backslash C\} \cup \{V(y) : y \in C\}$ is a m-open cover of Y, and it has a subcover {Y\C} ∪ {V(yi) : y_i ∈ C, 1 ≤ i ≤ n}. Let U = $\bigcap_{i=1}^{n}$ U(y_i) and $V = \bigcup_{i=1}^{n}$ V(y_i). It is easy to verify that C ⊂ V and (U × V) ∩ G(F) = ∅. Since U is m_{wg}-neighbourhood of x_0, U ∩ F^-(C) ≠ ∅. It follows that ∅ ≠ (U × C) ∩ G(F) ⊂ (U × V) ∩ G(F). Which is a contradiction. Hence the proof is completed.

Lemma:4.14

A multifunction F: (X, $\tau^* m_X$) → (Y, m_Y), has a strongly m_{wg}-closed if and only if for each $(x,y) \in (X \times Y)$-G(F),there exists $U \in m_X$WGO(X,x) and V∈ m_Y-WGO(Y,y)such that F(U)∩ $m_Y - Cl(V)$=∅.

Theorem: 4.15

If F: (X,$\tau^{*}m_{X}$) → (Y, m_{Y}) is upper $\tau^{*}m_{wg}$-continuous multifunction such that F(x) is $\tau^{*}m_{wg}$-compact for each $x \in X$ and Y is a m-Urysohn space, then G(F) is strongly $\tau^{*}m_{wg}$-closed.

Proof:

Let (x, y) ∈ (X × Y)−G(F), then y ∈ Y−F(x). Since Y is a $\tau^{*}m_{wg}$-Urysohn space, there exist $\tau^{*}m_{wg}$-open sets V and W of Y such that y ∈ V, F(x) ⊂ W and m_{Y}− Cl(V) ∩ m_{Y} − Cl(W) = ∅. Since F is upper $\tau^{*}m_{wg}$-continuous, there exists U∈ $\tau^{*}m_{X}$ -WGO(X, x) such that F (U) ⊂ m_{Y} − Cl(W). Then, we have F(U) ∩ m_{Y}-Cl(V) = ∅. Hence G(F) is strongly $\tau^{*}m_{wg}$-closed.

Theorem: 4.16

If F :(X,$\tau^{*}m_{X}$) → (Y,m_{Y}) is an upper $\tau^{*}m_{wg}$ -continuous multifunction and F(x) is m-compact. Also, let F(x)∩F(y)=∅ for each pair of $x, y \in X$ ($x \neq y$). If Y is m-Haussdroff space, then X is m-Urysohn space.

Proof:

Let F(x) ∩ G(x) = ∅ for each pair of $x, y \in X$ ($x \neq y$). Since Y is m-Haussdroff space, F(x) and F(y) are m-compact sets, there exist m-open sets V,W ⊂ Y such that $F(x) \subset V, F(y) \subset W$ such that V ∩ W = ∅. Since F is $\tau^{*}m_{wg}$-continuous multifunction, there exist U_1⊂ $\tau^{*}m_{X}$−WGO(X, x) and U_2⊂ $\tau^{*}m_{X}$−WGO(X, y) such that x ∈ $\tau^{*}m_{X}$− Cl(U_1)⊂ F^{+}(V), y ∈ $\tau^{*}m_{X}$ − Cl(U_2)⊂ F^{+}(W). Hence $\tau^{*}m_{X}$ − Cl(V) ∩ $\tau^{*}m_{X}$− Cl(W) = ∅. Then, X is m-Urysohn space.

Theorem: 4.17

If F, G : (X, $\tau^{*}m_{X}$) → (Y, m_{Y}) are upper $\tau^{*}m_{wg}$ -continuous multifunction into m- Urysohn space Y and for each $x \in X, F(x)$ and $G(x)$ are m-compact in (Y, m_{Y}), then U = { x∈ X: F(x) ∩ G(x) ≠ ∅} is wg-closed in (X, $\tau^{*}m_{X}$).

Proof:

Let $x \in X - $ A. Then F(x) ∩ G(x) = ∅. Since Y is m-Urysohn space, there exist m-open sets P and Q such that F(x)⊂ P, G(x)⊂ Q and Cl(P) ∩ Cl(Q) = ∅. Since F is upper $\tau^{*}m_{wg}$-continuous, there exists U_1⊂ $\tau^{*}m_{X}$−WGO(X, x) such that F(U_1) ⊂ Cl(P). Since G is upper $\tau^* m_{wg}$ -continuous, there exists $U_2 \subset \tau^* m_X$–WGO(X, x) such that F(U_2) ⊂ Cl(Q). Put U = $U_1 \cap U_2$, then we have U ∈ $\tau^* m_X$ -WGO(X, x) and U ∩ A = ∅. Hence, A is $\tau^* m_{wg}$ -closed in X.

Theorem: 4.18

If F: (X, $\tau^* m_X$) → (Y, m_Y) is an upper $\tau^* m_{wg}$ -continuous multifunction and $\tau^* m_{wg}$ -compact from a minimal space X to m-Urysohn space Y and let F(x) ∩ G(x) = ∅ for each x, y ($x \neq$ y) ∈ X. Then X is $\tau^* m_{wg}$ -Haussdroff space.

Proof:

Let x and y be any two distinct points in X. Then we have F(x) ∩ G(x)= ∅. Since Y is m-Urysohn space, there exist m-open sets P and Q such that F(x) ⊂ U, G(x) ⊂ V and m_Y-Cl(U) ∩ m_Y-Cl(V) = ∅. Since F is upper $\tau^* m_{wg}$ -continuous, then F(U) and F(V) are disjoint $\tau^* m_{wg}$ -open sets containing x and y respectively. Thus X is $\tau^* m_{wg}$ -Haussdroff space.

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